Multicommodity formulations for the prize collecting vehicle routing problem in the petrol industry

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Abstract

The Mobile Oil Recovery (MOR) unit is a truck designed to pump marginal oil wells in a petrol field. The MOR optimization Problem (MORP) consists in optimizing both the oil extraction and the travel costs. In this article, we describe several formulations for the MORP using a single vehicle and we propose two formulations to the case where several vehicles are used. We strengthen the proposed formulations by taking advantage of the MORP characteristics, by improving the number of subtour elimination constraints and by using cuts. Computational results are presented for instances close to the reality and optimality is proved for instances with up to 200 nodes.

Keywords: Vehicle routing problem, prize-collecting, selective traveling salesman, multiobjective.

Résumé

Les unités mobiles de pompage sont des camions munis d'un système d'extraction de pétrole. Ils sont utilisés pour les puits ayant une production marginale. Le problème d'optimisation des unités mobiles de pompage consiste à maximiser la quantité totale de pétrole extrait en définissant des tournées quotidiennes tout en minimisant le temps total de parcours. Dans ce travail, nous décrivons plusieurs formulations pour ce problème avec un véhicule unique et nous proposons deux formulations pour le problème avec une flotte de véhicules. Les formulations sont renforcées par l'introduction d'inégalités valides, mais aussi par l'utilisation des caractéristiques du problème pour réduire la quantité de contraintes d'élimination de sous-cycles. Des résultats numériques sont présentés pour des instances proches de la réalité. L'optimalité est prouvée pour des instances avec 200 sommets.

Mots clés : Tournées de véhicules, prize-collecting, problème du voyageur de commerce sélectif, multiobjectif.

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1 Introduction

Much effort has been made to increase the oil production in Brazil though the use of new technologies. As a consequence, the Brazilian oil production has met the country's need in 2006 and the country is globally self sufficient. The Rio Grande do Norte basin has been exploited for the last 30 years and about 98% of the oil wells are pumped using artificial lift systems. One such system is the Mobile Oil Recovery (MOR) unit.

The MOR unit is an artificial lift system which is used to exploit wells whose production is marginal. It consists of a truck (vehicle unit) equipped with an apparatus, see Figure 1. Considering a working day, the unit starts its tour at the depot, then it pumps several wells before returning to the depot. At each well, the driver spends some time to connect the unit to the well, to pump the oil and to disconnect the equipment. Whenever the unit's tank is full, an auxiliary vehicle is used to transfer the oil from the MOR unit to its own tank and to carry it back to the depot. Thus, the MOR unit does not need to stop its operations and its capacity can be considered unlimited.

Figure 1: An example of a MOR vehicle.

Typically, the level of oil within a well is not static: it raises over the time until its stabilization level is reached [4]. This happens when the pressure between the wells and the rock formation is stabilized. We consider here the oil level is static for all wells. It is computed by performing a statistic analysis of its history (last time it was pumped versus estimated production). The volume extracted from the wells is usually composed of a mixture of long-chain hydrocarbons (petroleo) and basic sediments and water (*bsw*). Thus, the "oil" production of a well referred in this work takes into account the oil volume and its quality, which depends on its *bsw* factor.

The MOR optimization Problem (MORP) is a multiobjective problem which consists in finding a set of wells to be pumped in a working day to maximize the oil extraction and to minimize the travel time. The two objectives are opposite, one pushing to increase profit and the other to reduce costs. With one MOR unit, the problem is close to the Selective Traveling Salesman Problem [15] which is also called Orienteering Problem, see e.g. [11], or

Maximum Collection Problem [13]. With a fleet of vehicles, the problem becomes a Vehicle Routing Problem (VRP) close to the Prize-Collecting VRP [5]. For further investigation on routing problems, readers are referred to the following works: the state of the art on exact and approximated methods for the VRP and its variants are found in [23] and an overview covering about 500 papers on classical routing problems are found in [16]. For multiobjective solutions strategies on routing problems, see [7, 9, 14].

A mathematical formulation for the MORP is proposed in [22] for a single MOR unit. A two-step optimization is performed: the oil extraction is maximized in the first phase. The second phase aims at minimizing both the travel and the operation time such that the oil extraction is not smaller than the value obtained in the first phase. Heuristics applications of the MORP are presented in [1, 22].

In this work, we propose several formulations for the MORP with a single vehicle and for a fleet of vehicles. Our contributions for the MORP with a single vehicle are: (i) remove redundant constraints, (ii) simplify the flow conservation constraints, (iii) test different strategies to eliminate invalid subtours, and (iv) strengthen the subtour elimination constraints. When a fleet of vehicles are used two formulations are proposed. The proposed formulations are strengthened by taking advantage of the MORP characteristics, by reducing the number of subtour elimination constraints, by introducing generalized bounds, and by adding cuts. Instances with up to 200 nodes, simulating practical situations, are solved. Moreover, it is shown that a formulation with a weaker linear relaxation can be very useful in practice, since it proves optimality for real size instances faster than the multiflow formulation. The first ever results for the MORP with several vehicles are presented in this work.

The work is organized as follows: the problem is defined in Section 2. Formulations for one unit are presented in Section 3. Sections 4.1 and 4.2 are devoted to the MORP with several units. Computational results are shown in Section 5 and final remarks are made in Section 6.

2 The MORP problem

The geographical data (roads, wells and depot) are modeled as an undirected graph $G = (N, E)$. G is preprocessed to build a complete digraph $G' = (V, A)$ where V is the set of wells and the depot v_0. Let d_{ij} be the shortest distance from i to j, $\forall(i, j) \in A$, and let s be the MOR unit average speed. Thus, for every arc of G', the travel time t_{ij} is computed as $t_{ij} = d_{ij}/s$.

The process of dynamically filled wells was studied in [4]. Let H^i be the static level for well i and let us assume that well i is filled with a factor τ^i. Wells are filled until the static level is reached, according to Equation (1), where $h^i(t)$ is the oil level within well i at instant t. The bigger τ^i, the slower well i is filled. We consider that wells can be exploited any time even if the static level is not reached and that the MOR units can visit a well only one time a day as in the previously works [6, 19, 21]. In fact, the proposed formulations do not deal with dynamically filled wells. A production estimation based on the well history is calculated each day for each well and this value is used instead. Thus, the oil level is considered static.

$$h^i(t) = H^i \cdot (1 - e^{-t/\tau^i}) \qquad \forall i \in V \tag{1}$$

Let t'_i be the total operation time at well i (time to connect the MOR unit, to pump, and to disconnect the unit). Denote by $p_i = v_i \cdot (1 - bsw_i)$ the oil production of well i (its prize), where bsw_i is the amount of basic sediments and water, and when v_i is the estimated volume to be extracted. Let P and T be respectively the total oil production and the total time of a MOR unit in a working day. Moreover, $\overline{T}$ is the maximal time a MOR unit can work in a day.

Given K, the total number of MOR units, the MORP consists in defining one circuit $\tau = \{v_0, v_{\sigma 1}, v_{\sigma 2}, ..., v_{\sigma k}, v_0\}$ for each MOR unit, where σ is the position of wells in the circuit to be exploited in a day, such that P is maximized and T is minimized. The time limit $T \leq \overline{T}$ has to be satisfied.

As mentioned before, the MORP with a single vehicle is close to two classical problems, the selective traveling salesman problem (STSP) and the prize collecting traveling salesman problem (PCTSP). In the STSP, a prize is associated to each node visited. The problem consists in defining a circuit (the tour starts and ends at a fixed node, e.g., the depot) such that the total prize is maximized (this corresponds to the oil production for the MORP), and such that the tour does not exceed a given length (this corresponds to the working time in the MORP) [15]. The main difference between MORP and STSP is that the MORP also requires to minimize. Furthermore, in the MORP there are also an operation time associated to each node visited. In PCTSP, a prize is given for each visited node and a penality is paid for each unvisited node. A minimum level prize must be reached in a tour. The objective is to define a tour such that the travel cost and the penalities are minimized [5]. The differences between the PCTSP and the MORP are: instead of paying a penality for each unvisited node, a penality (operation time) is paid for each visited node in the MORP. The objective function for the MORP has opposite goals: the primary objective pushes to collect prizes (the oil production) and the secondary aims at reducing costs. The MORP with several vehicles is close to a generalization of the PCTSP with a fleet of vehicles, denoted here of the prize-collecting VRP (PCVRP).

3 Formulations using a single vehicle

As far as we know, only one formulation has been proposed in the literature to the MORP [2, 3]. In this formulation, one vehicle is considered and the optimization is done in two phases: first, compute the maximal amount of oil (prize) that can be extracted in a working day and second, compute the shortest route to extract this amount. We propose a formulation for the MORP with a single vehicle based on the formulation proposed in [22]. The main differences rely on: we have removed the constraints ensuring the MOR unit returns to the depot since (constraints 5 and 6 guarantee the return to the depot). The flow conservation constraints were simplified. We have also tested several subtour elimination strategies. In [22], only the Gavish and Graves strategy to eliminate subtour was used. Moreover, an upper bound on the number of wells that can be exploited in a working day is defined. This bound is used to strengthen the proposed formulation.

Let $f_{ij} \in \{0, 1\}$ be the decision variable on the choice of arc (i, j) and let x_i be the binary variables which specify if well i is exploited or not. The first optimization phase for the MORP is given as follows:

$$\max \ P = \sum_{i \in V \setminus \{v_0\}} p_i \cdot x_i \qquad s.t. \tag{2}$$

$$\sum_{i \in V \setminus \{v_0\}} t'_i \cdot x_i + \sum_{(i,j) \in A} t_{ij} \cdot f_{ij} \leq \overline{T} \tag{3}$$

$$\sum_{j:(j,i) \in A} f_{ji} - \sum_{j:(i,j) \in A} f_{ij} = 0 \quad \forall i \in V \setminus \{v_0\} \tag{4}$$

$$\sum_{j:(j,i) \in A} f_{ji} = x_i \quad \forall i \in V \setminus \{v_0\} \tag{5}$$

$$\sum_{j \in V} f_{0j} = 1 \tag{6}$$

$$\text{(subtour eliminations constraints)} \tag{7}$$

$$x_i \in \{0, 1\} \qquad \forall i \in V \tag{8}$$

$$f_{ij} \in \{0,1\} \qquad \forall (i,j) \in A \tag{9}$$

The objective function (2) aims at minimizing the oil extraction. Inequality (3) limits the working time (travel and operation time) to $\overline{T}$. The flow conservation constraints are (4) and (5). Restriction (6) guarantees the tour starts at the depot. Variables x_i and f_{ij} are respectively defined in Constraints (8) and (9). We discuss in Section 3.1 the use of several subtour elimination constraints: those of Miller and Tucker and Zemlin (MTZ) [10, 18], and those of Gavish and Graves using either aggregated (GGA) or disaggregated flow (GGD) [12]. GGA constraints are used in [22].

The objective of the second optimization phase is to minimize the working time (10) subject to Constraints (4)–(9) and (11). Constraint (11) restricts the total production to be equal to the total optimal prize P^* obtained in the first phase.

$$\min \; T = \sum_{i \in V \setminus \{v_0\}} t'_i \cdot x_i + \sum_{(i,j) \in A} t_{ij} \cdot f_{ij} \qquad s.t. \tag{10}$$

$$\sum_{i \in V \setminus \{v_0\}} p_i \cdot x_i = P^* \tag{11}$$

Constraints (4)–(9).

3.1 Subtour eliminations constraints

A subtour is defined by any ordered subset of vertices. For the MORP, only subtours starting and ending at the depot v_0 are valid. Subtour constraints have been evaluated in the literature for the TSP problem, see e.g. [24]. MTZ, GGA and GGD subtour elimination constraints for the MORP, and some improvements are described below.

An upper bound on the number M of wells that can be exploited in a working day can be computed. Considering the working time of the MOR unit, a simple procedure consists in computing M by sorting the wells in increasing order of operation time t'_i [22]. Thus, M is such that:

$$\sum_{i=1}^{M} t'_i \leq \overline{T} \leq \sum_{i=1}^{M+1} t'_i. \tag{12}$$

We propose to strengthen the value of M by taking also into account the travel time using the following argument: to arrive at a node, the vehicle has to use an arc. Thus, the minimum travel time to arrive at each node can be considered as follows:

$$\sum_{i=1}^{M} t'_i + \{\underset{j \in V}{min}\, t_{ji}\} \leq \overline{T} \leq \sum_{i=1}^{M+1} t'_i + \{\underset{j \in V}{min}\, t_{ji}\} \tag{13}$$

Another argument can be used to strengthen even more M. Since the vehicle must return to the depot, the minimum travel time to arrive at the depot can also be considered. Then, M is such that:

$$\sum_{i=1}^{M} t'_i + \{\underset{j \in V}{min}\, t_{ji}\} \leq \overline{T} - \{\underset{j \in V}{min}\, t_{j0}\} \leq \sum_{i=1}^{M+1} t'_i + \{\underset{j \in V}{min}\, t_{ji}\} \tag{14}$$

3.1.1 Lifted Miller, Tucker and Zemlin Constraints.

The Miller, Tucker and Zemlin constraints define a topological order to eliminate invalid subtours. However, for the MORP, the depot appears twice (at the beginning and at the end). Thus, one can duplicate the depot and work on a support graph. We consider instead the depot only at the beginning of the topological design. This can be done since the flow structure defined by variables f_{ij} and x_i, and Constraints (4)–(6) guarantees the return to the depot.

The corresponding MTZ constraints for the MORP is given in Equations (15)–(16). Variable u_i states the order well i appears in the solution, thus defining a topological order.

$$u_i - u_j + |V| \cdot f_{ij} \leq |V| - 1 \quad \forall (i,j) \in A, j \neq \{v_0\} \tag{15}$$

$$0 \leq u_i \leq |V| - 1 \qquad \forall i \in V \tag{16}$$

There are $O(|V|^2)$ of such MTZ constraints, which can be strengthened by using M as computed in 14, instead of $|V|$.

$$u_i - u_j + M \cdot f_{ij} \leq M - 1 \qquad \forall (i,j) \in A, j \neq \{v_0\} \tag{17}$$

$$0 \leq u_i \leq M \qquad \forall i \in V \tag{18}$$

These constraints can be lifted even more using the same ideas as Desrochers and Laporte [10]. The idea of lifting consists in adding a valid non-negative term $\alpha_{ji} f_{ji}$ to the Inequalities (17)

$$u_i - u_j + M \cdot f_{ij} + \alpha_{ji} \cdot f_{ji} \leq M - 1 \qquad \forall (i,j) \in A, j \neq \{v_0\} \tag{19}$$

The larger the value of α_{ji}, the stronger the reduction in the original solution space. If $f_{ji} = 0$, then α_{ji} may take any value. Suppose now $f_{ji} = 1$. Then, the MOR unit exploits well $j \neq v_0$ before well i, $u_i = u_j + 1$. Thus, $f_{ji} = 1$ implies $f_{ij} = 0$, otherwise there is a subtour (i, j). By substitution, we obtain $\alpha_{ji} \leq M - 2$. The larger α_{ji}, the stronger is the lift. Thus, $\alpha_{ji} = M - 2$. A lifted version of Constraints (17) is given in Inequalities (20).

$$u_i - u_j + M \cdot f_{ij} + (M-2) \cdot f_{ji} \leq M - 1 \quad \forall (i,j) \in A, j \neq v_0 \tag{20}$$

3.1.2 Gavish and Graves Constraints.

The Gavish and Graves [12] approach removes invalid subtours by building a network flow. A flow is sent to the nodes of the tour. Each node consumes one unit. In disaggregated flow, a specific flow is sent from the source to each node [8, 17]. Otherwise, if the flow is not specified, it is an aggregated flow.

Let y_{ij} be the flow variable on arc (i, j). Thus, GGA constraints for the MORP are given in Equations (21)–(23). Constraints (21) are the flow conservation constraints. Inequalities (22) state a flow uses the arc (i, j) if it is selected. These constraints are strengthened by using M. In this strategy, there are $O(|V|^2)$ constraints and variables.

$$\sum_{j:(j,i)\in A} y_{ji} - \sum_{j:(i,j)\in A} y_{ij} = x_i \quad \forall i \in V \backslash \{v_0\} \tag{21}$$

$$y_{ij} \leq M \cdot f_{ij} \qquad \forall (i,j) \in A, \tag{22}$$

$$y_{ij} \geq 0 \qquad \forall (i,j) \in A \tag{23}$$

The GGD version is given in Constraints (24)–(27). Let y_{ij}^l be the variable specifying if flow for node l traverses arc (i, j) or not. Constraints (24) are the flow conservation constraints. Equations (25) state that a flow unit is sent from the source to each node l. Restrictions (26) specify that flow for node l traverses arc (i, j) if and only if it is used.

$$\sum_{j:(i,j)\in A} y_{ij}^l - \sum_{j:(j,i)\in A} y_{ji}^l = 0 \qquad \forall l \in V \backslash \{v_0\}, \forall i \in V \backslash \{v_0, l\} \tag{24}$$

$$\sum_{j:(0,j)\in A} y_{0j}^l = x_l \qquad \forall l \in V\backslash\{v_0\} \tag{25}$$

$$y_{ij}^l \leq f_{ij} \qquad \forall l \in V\backslash\{v_0\}, \forall (i,j) \in A \tag{26}$$

$$y_{ij}^l \geq 0 \qquad \forall l \in V\backslash\{v_0\}, \forall (i,j) \in A \tag{27}$$

This strategy implies $O(|V|^3)$ of such constraints and variables and it produces better linear relaxation than using the aggregated flow (21)–(23). However, we show in the computational experiments that under some conditions, the aggregated flow strategy can produce a better linear relaxation as Constraints (22) are stronger with a lifted value for M.

4 Formulations using several vehicles

In this section, we present two formulations for the MORP using several MOR units. The first one is a three-indexed formulation. The second formulation is more compact. Both formulations are designed for the two optimization phases as presented in Section 3.

4.1 A three-indexed formulation using several vehicles

In the three-indexed formulation, we specify explicitly which MOR unit exploits each well. Thus, new variables are defined as follows: let x_i^k be a decision variable that specifies if well i is exploited by the vehicle k or not. Variables $f_{ij}^k \in \{0,1\}$ state if vehicle k exploits well j after well i or not. $P(K)$ is the total profit collected using the K MOR units. All other terms are defined in Section 3. The three-indexed formulation is as follows:

$$\max \;\; P(K) = \sum_{i\in V\backslash\{v_0\}} p_i \cdot \sum_{k=1}^{K} x_i^k \qquad s.t. \tag{28}$$

$$\sum_{i\in V\backslash\{v_0\}} t_i' \cdot x_i^k + \sum_{(i,j)\in A} t_{ij} \cdot f_{ij}^k \leq \overline{T} \qquad \forall k = 1, ..., K \tag{29}$$

$$\sum_{j:(j,i)\in A} f_{ji}^k - \sum_{j:(i,j)\in A} f_{ij}^k = 0 \qquad \forall k = 1, ..., K, \forall i \in V\backslash\{v_0, k\} \tag{30}$$

$$\sum_{j:(0,j)\in A} f_{0j}^k \leq 1 \qquad \forall k = 1, ..., K \tag{31}$$

$$\sum_{j:(j,i)\in A} f_{ji}^k = x_i^k \qquad \forall k = 1, ..., K,\ \forall i \in V\backslash\{v_0\} \tag{32}$$

$$\sum_{k=1}^{K} x_i^k \leq 1 \qquad \forall i \in V\backslash\{v_0\} \tag{33}$$

$$\sum_{j:(j,i)\in A} y_{ji} - \sum_{j:(i,j)\in A} y_{ij} = \sum_{k=1}^{K} x_i^k \qquad \forall i \in V\backslash\{v_0\} \tag{34}$$

$$y_{ij} \leq M \cdot \sum_{k=1}^{K} f_{ij}^k \qquad \forall (i,j) \in A, j \neq v_0 \tag{35}$$

$$y_{ij} \geq 0 \qquad \forall (i,j) \in A \tag{36}$$

$$x_i^k \in \{0,1\} \qquad \forall k = 1, ..., K, \forall i \in V\backslash\{v_0\} \tag{37}$$

$$f_{ij}^k \in \{0,1\} \qquad \forall k = 1, ..., K, \forall (i,j) \in A \tag{38}$$

Restrictions (29) limit the units work in a day. The flow conservation constraints are defined in (30) and (31). Constraints (32) ensure that unit k pass though an arc (i,j) only if it exploits well i. Inequalities (33) specify that at most one unit visits well i. Constraints (34) and (35) are the GGA subtour elimination constraints. Constraints (36)–(38) are the variables definition. This formulation contains $O(|V^3|)$ variables and constraints and can be seen as a generalization of formulation (2)–(9) to several vehicles. The GGA constraints are chosen according to the computational results for one vehicle (Section 5). Obviously, other strategies could be used as well.

4.1.1 Valid inequalities for the three-indexed formulation

In the previous model, any vehicle can be assigned to any route in the solution. Thus, the number of feasible solutions are multiplied by up to K (ways to assign K vehicles to the K routes). This can dramatically slow down the effectiveness of the model. Figure 2 illustrates the symmetry problem. Figure 2 (a) and (b) have the same tours, but in Figure 2 (a) vehicle k_1 does the tour $\{v_0, v_3, v_6, v_7, v_1, v_0\}$ and in the Figure 2 (b), it is done by the vehicle k_2. The same applies to the second tour. Thus, the idea is to use valid inequalities to avoid these situations.

To remove the symmetry, so-called symmetry-breaking constraints can be defined. A first way is to specify that the first vehicle does the tour with the

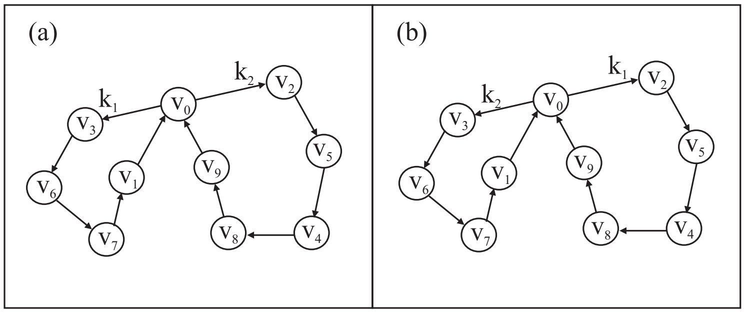

Figure 2: An example of symmetry between two solutions.

highest oil production, the second one does the tour with the second highest oil production, and so on. This is done through Inequalities (39).

$$\sum_{i \neq v_0} p_i x_i^k \geq \sum_{i \neq v_0} p_i x_i^{k+1} \qquad \forall k = 1, ..., K-1 \tag{39}$$

Another way to remove symmetry consists in applying a lexicographical order on the routes: given two circuits τ_1 and τ_2, $\tau_1 < \tau_2$ if the first node visited in τ_1 has a lower identifier than the first node visited in τ_2. Constraints (40) define such a lexicographical order.

$$\sum_{i \neq v_0} i \cdot x_{0i}^k \leq \sum_{i \neq v_0} i \cdot x_{0i}^{k+1} \qquad \forall k = 1, ..., K-1 \tag{40}$$

The lexicographical order can be defined another way as in Constraints (41). The sum of the first n' variables on the outgoing arcs is greater in the first circuits.

$$\sum_{i=1}^{n'} x_{0i}^k \geq \sum_{i=1}^{n'} x_{0i}^{k+1} \qquad \forall k = 1, ..., K-1, \forall n' < n \tag{41}$$

Finally, the last way to remove symmetry is to use Constraints (42). Vehicle k goes to node i only if vehicle $k-1$ goes to at least one of the nodes $1, ..., i-1$). Consequently, all variables x_i^k such that $k > i$ can be set to zero.

$$x_i^k \leq \sum_{j=1}^{i-1} x_j^{k-1} \qquad \forall i = 1, ..., n, \forall k = 2, ..., \min\{K, i\} \tag{42}$$

4.2 A two-indexed formulation using several vehicles

In the two-indexed formulation, we do not explicitly define which unit exploits well i since the MOR units have the same characteristics (homogeneous fleet). A similar idea was previously used in the literature, for example, in [20]. Instead of explicitly assigning a MOR unit k to a subset of wells, a restriction is set on the date (time) well i is exploited using a unit, whatever it is.

We consider the variables f_{ij} and x_i as defined in the Section 3 for the single vehicle formulation. Additionally, variables d_i specify the date (time) well i is visited by a MOR unit in a working day.

$$\max \; P = \sum_{i \in V \backslash \{v_0\}} p_i \cdot x_i \qquad s.t. \tag{43}$$

$$\sum_{j:(j,i) \in A} f_{ji} - \sum_{j:(i,j) \in A} f_{ij} = 0 \qquad \forall i \in V \backslash \{v_0\} \tag{44}$$

$$\sum_{j:(j,i) \in A} f_{ji} = x_i \qquad \forall i \in V \backslash \{v_0\} \tag{45}$$

$$\sum_{j \in V} f_{0j} = K \tag{46}$$

$$d_i - d_j + (\overline{T} + t'_i + t_{ij}) \cdot f_{ij} + (\overline{T} - t'_j - t_{ji}) \cdot f_{ji} \leq \overline{T} \qquad \forall (i,j) \in A, i, j \neq v_0 \tag{47}$$

$$d_i \geq t_{0i} \cdot f_{0i} + \sum_{j \neq v_0} (t_{0j} + t'_j + t_{ji}) \cdot f_{ji} \qquad \forall i \in V \backslash \{v_0\} \tag{48}$$

$$d_i \leq \overline{T} - (t'_i + t_{i0}) \cdot f_{i0} - \sum_{j \neq v_0} (t'_i + t_{ij} + t'_j + t_{j0}) \cdot f_{ij} \qquad \forall i \in V \backslash \{v_0\} \tag{49}$$

$$x_i \in \{0, 1\} \qquad \forall i \in V \tag{50}$$

$$f_{ij} \in \{0, 1\} \qquad \forall (i,j) \in A \tag{51}$$

$$d_i \geq 0 \qquad \forall i \in V \backslash \{v_0\} \tag{52}$$

The flow conservation is given in Constraints (44). Restrictions (45) ensure arc (j, i) is used if well i is exploited. Equations (46) state all the

K MOR units are used. Constraints (47) link the time node j is visited, to the time node i is visited, and to the selection of arc (i, j). This is an adaptation of the lifted MTZ constraints (see Section 3.1). Inequalities (48) and (49) define generalized lower and upper bounds on the time node i is visited. Inequalities (48) link the time node i is visited to variables f_{ji}. At most one of the arcs entering node i is used. Thus, d_i is at least equal to the minimal time required to arrive at node i, either by going from v_0 to i or by going from j to i. The same idea applies to the Inequalities (49). Variables x_i^k, f_{ij}^k and d_i are defined respectively by Constraints (50) to (52). The two-indexed formulation has $O(|V|^2)$ variables and constraints. MTZ is used since the time constraints definition is straightforward and the formulation has still $O(|V|^2)$ variables.

5 Computational results

The computational experiments were carried out on an Intel Core 2 Duo with 2.66 GHz clock and 4Gb of RAM memory, using CPLEX 11 under default parameters. Instances were generated using a geographical information system to simulate real situations. Comparison among the proposed formulations are measured in terms of time to prove optimality and of linear relaxation.

In the Tables 1 and 2, each line corresponds to an instance. For each instance, the working day length (L) in minutes, the number of wells ($|V|$) and its optimal production (P*) are given. For each formulation, the linear relaxation value (RL*), the time (T) spent by the unit in the optimal solution, the time (time(s)) required to prove optimality in seconds (rounded up), and the number of nodes (nodes) explored in the branch-and-bound tree are presented. The symbol $(-)$ means the solver did not prove optimality because it ran out of memory. When the optimal solution is unknown, the best integer solution found so far is identified by "($\geq$ value)".

Table 1 summarizes the results for the formulations for one vehicle using the MTZ, GGA or GGD subtour elimination constraints. From the computational results, GGA proves optimality faster than MTZ and GGD for 9 instances. MTZ proves optimality faster than GGA and GGD for 7 instances. In spite of having the worst linear relaxation, MTZ is able to prove optimality for instances with up to 200 nodes. GGD consumes a lot of time even if it produces good linear relaxation. An interesting result on the linear relaxation is found for $L = 480$ and $|V| = 20$: the GGA linear relaxation is better than the linear relaxation of GGD. This happens here because the value of M is equal to the optimal amount of wells exploited in a day.

We performed an experiment to illustrate the evolution of the GGA linear relaxation values when M varies. We compare it with the GGD linear relaxation values. Results are presented for the instance with $|V| = 20$ wells in Figure 3. The x and y axis represent respectively the values of M, and the oil production. For this instance, the MOR unit can exploit at most $M = 4$ wells. The results show that the smaller M the tighter the linear relaxation of GGA. For some instances, in terms of linear relaxation, a value of M close to the optimal amount of wells exploited in a day is sufficient to make GGA better than GGD can even help GGA having better linear relaxation than GGD.

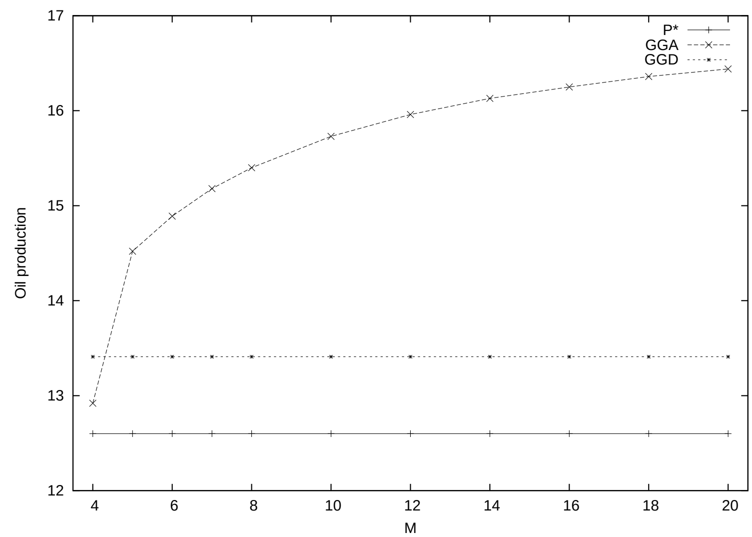

Figure 3: Evolution of the linear relaxation for the formulations GGA and GGD.

We have also run the second optimization phase for all instances presented in Table 1. The time T was only improved for the instance with $L = 480$ and $|V| = 120$ ($T^* = 479.5$ instead of $T = 480$). Thus, results of the second optimization phase were not tested for several vehicles. Even so, it remains valuable since it takes place in the global decision process of the problem.

The results for the two-indexed and the three-indexed formulations are presented in Table 2. The number of vehicles used (K) and the sum of the total time spent by all the MOR units (T') are given. The three-indexed formulation produces a better linear relaxation than the two-indexed formulation. However, the two-indexed formulation performs better to compute the optimal solution. In addition to the number of wells, the problem becomes more difficult when the number of vehicles increases. Moreover, the working day limit also contributes to the difficulty of the problem. Results suggest

Table 1: The first optimization phase for the MORP using one vehicle.

			MTZ				GGA				GGD			
L	$\|V\|$	P*	RL*	T	time	nodes	RL*	T	time	nodes	RL*	T	time	nodes
480	20	12.60	17.21	477	2	1292	12.92	477	0	1	13.41	477	11	17
480	30	15.88	18.38	477	5	2337	17.19	477	6	979	16.61	477	841	191
480	40	15.88	18.38	477	7	2081	17.19	477	6	963	16.61	477	30137	255
480	60	15.84	18.43	467	15	3203	16.86	467	3	90	-	-	-	-
480	80	9.97	13.80	479	83	5429	11.67	479	32	1410	-	-	-	-
480	120	18.73	19.09	480	73	2996	19.05	480	204	1910	-	-	-	-
480	160	19.10	19.47	480	240	2095	19.46	480	2540	3378	-	-	-	-
480	200	19.64	19.82	480	62	3256	19.77	480	20626	2360	-	-	-	-
960	20	24.45	32.11	952	817	915570	28.39	952	98	14807	25.65	960	452	264
960	30	31.65	35.93	950	424	228949	35.29	950	446	46898	32.43	950	5313	527
960	40	19.76	24.17	909	406	40244	23.84	909	135	10898	22.34	909	63631	2401
960	60	31.65	35.96	950	974	301668	35.18	950	377	33257	32.26	-	-	-
960	80	37.70	38.05	959.5	3240	57320	38.00	959.5	866	21595	-	-	-	-
960	120	37.99	38.41	960	2764	36803	38.42	960	872	6716	-	-	-	-
960	160	40.05	40.19	960	377	5893	40.16	960	585	877	-	-	-	-
960	200	40.05	40.19	960	420	6672	40.16	960	789	951	-	-	-	-

Table 2: The first optimization phase for the MORP using several vehicles.

				Two-indexed formulation				Three-indexed formulation			
K	L	$\lvert V \rvert$	P^*	RL*	T'	time (s)	nodes	RL*	T'	time (s)	nodes
2	480	10	20.97	27.74	911	2	8416	24.09	**870**	22	9139
2	480	20	24.45	29.55	953	74	41047	25.08	953	4	487
2	480	30	31.16	35.79	941	51	51661	33.78	941	1149	75216
2	480	40	31.16	35.79	941	646	84815	33.78	941	1315	64397
2	480	50	28.99	34.70	928	654	94922	30.93	928	7929	101935
2	480	60	30.39	35.78	946	326	57405	32.99	946	1619	60724
2	480	70	25.88	32.86	916	2	3374	30.43	916	1219	26108
2	480	80	19.35	27.03	881	78	69927	22.86	**876**	16426	313421
2	960	20	$\geq$ 46.51	55.89	-	-	-	52.75	-	-	-
2	960	30	$\geq$ 62.26	69.44	-	-	-	68.57	-	-	-
3	480	10	29.82	29.82	909	1	503	29.82	**901**	2	639
3	480	20	33.72	40.42	943	553	704266	36.73	943	27788	214856
3	480	30	45.49	52.31	943	2394	2269664	49.86	-	-	-
3	960	20	62.16	62.16	933	134	138791	62.16	-	-	-
3	960	30	$\geq$ 88.78	98.44	-	-	-	97.63	-	-	-

it is suitable to use a small time window (480 minutes). The three-indexed formulation found sometimes a smaller value of T' as shown in bold.

6 Concluding remarks

Several formulations for the MORP are proposed in this work and the first ever results using several vehicles are presented. Additionally, we proposed to improve the subtour constraints by taking advantage of the time window. Thus, instances close to reality (up to 200 wells) are solved. Among the formulations for one vehicle, GGA performs globally better than MTZ and GGD to prove optimality. For several vehicles, the two-indexed formulation is faster to prove optimality in spite of weaker linear relaxations.

Computational experiments show that the time window restriction plays a key role in computing an optimal solution: the smaller the time window, the easier the problem to solve. Optimal solutions can be computed for medium-sized instances with two MOR units. When using three vehicles, this does not hold as the CPU time increases dramatically for small instances.

The larger instances used here are larger than the problems considered by the company in the Rio Grande do Norte Basin. Consequently, the oil company is now able to compute the optimal solution for the MORP instead of using solutions given by heuristics.

For future work, there are several promising research to be developed for this problem. For example, investigating situations where the second optimization phase becomes really useful. Moreover, the experiments show that for large time windows the problem becomes more difficult. Thus, we could study an approach to split large time windows. In terms of algorithms, several strategies could be explored such as exact algorithms as a branch-and-cut. It could be applied for difficult cases when a large number of vehicles are needed. Furthermore, pareto-based strategies to deal with the multiobjective function can be tested. Finally, the dynamic oil filling in the well can be integrated the models.

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